

Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed
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This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

===== [Exercise 6773]

Dissipation phase space volume and entropy

Consider the derivation of Liouville’s theorem for the ensemble density $\rho(p, q, t)$ in phase space (p, q) corresponding to the motion of a particle of mass m with friction γ

$$\frac{dq}{dt} = \frac{p}{m}, \quad \frac{dp}{dt} = -\gamma p.$$

- (a) Show that Liouville’s theorem is replaced by $d\rho/dt = \gamma\rho$.
- (b) Assume that the initial $\rho(p, q, t=0)$ is uniform in a volume ω_0 in phase space and zero outside of this volume. Find $\rho(p, q, t)$ if ω_0 is a rectangle $-\bar{p} < p < \bar{p}$, $-\bar{q} < q < \bar{q}$. Find implicitly $\rho(p, q, t)$ for a general ω_0 .
- (c) what happens to the occupied volume ω_0 as time evolves? (assume a general shape of ω_0). Explain at what t this description breaks down due to quantization.
- (d) Find the Boltzmann entropy as function of time for case (b). Discuss the meaning of the result.