Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

[Exercise 6772]

Boltzmann equation: distribution function

Consider an ideal gas in an external potential $\phi(\mathbf{r})$.

(a) Let $H = \int d^3v \int d^3r f(\mathbf{r},\mathbf{v},t) \ln f(\mathbf{r},\mathbf{v},t)$
where $f(\mathbf{r},\mathbf{v},t)$ is arbitrary except for the conditions on density n and energy E

$$\int d^3r \int d^3v f(\mathbf{r},\mathbf{v},t) = n \ \ , \ \ \int d^3r \int d^3v \left[\tfrac{1}{2}mv^2 + \phi(\mathbf{r})\right] f(\mathbf{r},\mathbf{v},t) = E\,.$$

Find $f(\mathbf{r},\mathbf{v})$ (i.e. t independent) which maximizes H. (Note: do not assume binary collisions, i.e. the Boltzmann equation).

(b) Use Boltzmann's equation to show that the general form of the equilibrium distribution of the ideal gas (i.e. no collision term) is $f[\frac{1}{2}mv^2 + \phi(\mathbf{r})]$ where the local force is $\nabla\phi$. Determine this solution by allowing for collisions and requiring that the collision term vanishes. Find also the average density $n\,(r)$.