Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

======= [Exercise 6080]

Einstein relation for the conductivity of electrons

Given a metal design. We mark with $\varphi(x)$ the electrical potential in the sample and with $N(x)$ the spacial density of the electrons in the design. According to the kinetic theory s

$$\vec{J}(x) = -\sigma\nabla\varrho - eD\nabla N$$

σ is the conductivity and D is the diffusion coefficient. In an equilibrium state $\vec{J}(x) \equiv 0$, especially in a state of equilibrium that we get in the presence of outer field $\varphi(x) \neq const$ and therefore has to : $\frac{\sigma}{eD} = \frac{\nabla N}{\nabla\varphi}$.
Use the principles of the statistical mechanics to show that from here derives

$$\frac{\sigma}{D} = -e^2 \int dE\ g(E) f'(E-\mu)$$

low temperatures $\frac{\sigma}{D} = e^2 g(E_r)$
High temperatures $\frac{\sigma}{D} = N\frac{e^2}{T}$
$g(E)$ is the uniparticles states density per volume unit. Hint - notice that

$$N(x) = \int g(E - eV(x))\, dE f(E-\mu)$$