E6070: Landauer formula for 1D conductance

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The problem:

Consider 1D conductor that has transmission coefficient g. The conductor is connected to 1D leads that have chemical potentials μ_a and μ_b. Assume $\mu_a = \mu$ and $\mu_b = \mu + eV$, where V is the bias.
(1) Write the expression for the current I as an integral over the occupation function $f(\epsilon)$.
(2) For small bias write the relation as $I = GV$ and obtain an expression for G. Write explicit results for zero temperature Fermi occupation (Landauer formula) and for high temperature Boltzman occupation.
(3) Find expressions for $I(V)$ in the case of arbitrary (possibly large) bias, for zero temperature Fermi occupation and for high temperature Boltzmann occupation. Assume that g is independent of energy

The solution:
(1) In general for $1D$ we have:

$$I_{a,n} = \frac{e}{L} v_{a,n}$$

Where $v_{a,n}$ is the speed of the electron in the cannel n.

$$I_a = \sum_{n=0}^{\infty} f_a(E_n - \mu_a) I_{a,n} = \int_0^{\infty} f_a(E - \mu_a) \frac{LdE}{2\pi v_a} (\frac{ev_a}{L}) = \frac{e}{2\pi} \int_0^{\infty} f_a(E - \mu_a) dE$$

Multiplying by the transition coefficient between a and b - g we get the total current:

$$I = g(I_b - I_a) = \frac{e}{2\pi} \int_0^{\infty} g(f_b(E - \mu_b) - f_a(E - \mu_a)) dE$$

(2) We now make use of the fact that the bias is small.

$$I = \frac{e}{2\pi} \int_0^{\infty} g(f_b(E-\mu-eV) - f_a(E-\mu)) dE \approx -\frac{eg}{2\pi} \int_0^{\infty} f'(E-\mu) eV dE = -\frac{e^2 g}{2\pi} V(f(\infty) - f(0)) = \frac{e^2 g}{2\pi} f(0) V$$

In the fermi case the occupation function is $f = \frac{1}{e^{\beta(E-\mu)}+1}$ therefore:

$$f(0) = 1 \implies G = \frac{e^2 g}{2\pi}$$

Which is the landauer formula.
For $T > 0$ we have the Boltzman occupation function $f = e^{-\beta(E-\mu)}$

$$f(0) = e^{\beta\mu} \implies G = \frac{e^2 g}{2\pi} e^{\beta\mu}$$

(3) In the case of an arbitrary bias we have:

$$I = \frac{e}{2\pi} \int_0^{\infty} g(f_b(E - \mu_b) - f_a(E - \mu_a)) dE$$

Using the fact that in $T = 0$ the fermi a occupation function becomes a step function we get:

$$I(V) = \frac{e}{2\pi} g(\mu_b - \mu_a) = \frac{e^2 g}{2\pi} V$$

And in the Boltzman case $T > 0$:

$$I(V) = \frac{eg}{2\pi} \int_0^{\infty} (e^{-\beta(E-\mu_b)} - e^{-\beta(E-\mu_a)}) dE = \frac{eg}{2\pi\beta} (e^{\beta\mu_b} - e^{\beta\mu_a}) = \frac{eg}{2\pi\beta} e^{\beta\mu} (e^{\beta eV} - 1)$$