E6040: Electron gas with magnetic field

Submitted by: Yaron de Leeuw

The problem:

A box with electrons of mass m is subjected to a magnetic field B. The single particle interaction is described by $-\gamma B \sigma_z$. The chemical potential of the electrons inside the box is μ. A hole through one of the walls is drilled. The electrons that are emitted from the hole with a velocity in the range $v < v' < v + dv$ are filtered, and subsequently their spin is measured. The measured current is defined as $I = I_\uparrow + I_\downarrow$.

(a) Find the ratio $\alpha(B;\mu) = (I_\uparrow - I_\downarrow)/I$.

(b) Find a linear approximation for $\alpha(B;\mu)$ regarded as a function of the magnetic field.

(c) What is the maximal value of $\alpha(B;\mu)/B$, and what is the range for which the result is valid.

The solution:

The flux for N electrons in volume V with velocity v is :

$$J = \iint_{|\theta|<\frac{\pi}{2}} \frac{d\Omega}{4\pi}\frac{N}{V} v\cos\alpha = \frac{1}{4}\left(\frac{N}{V}v\right)$$

The number of spin up electrons in the velocity range $v < v' < v + dv$ is given by the Fermi distribution:

$$N_\uparrow(v < v' < v + dv) = g(\frac{1}{2}mv^2 - \gamma B) f(\frac{1}{2}mv^2 - \gamma B - \mu)$$

So the current of spin up electrons, with velocity $v < v' < v + dv$ is:

$$I = J\delta A = g(\frac{1}{2}mv^2) f(\frac{1}{2}mv^2 - \gamma B - \mu)\frac{1}{4}v\delta A$$

The spin down current is:

$$I = J\delta A = g(\frac{1}{2}mv^2) f(\frac{1}{2}mv^2 + \gamma B - \mu)\frac{1}{4}v\delta A$$

We can now find $\alpha(B;\mu)$, the normalized current difference:

$$\begin{aligned}
\alpha(B;\mu) &= \frac{f(\frac{1}{2}mv^2 - \gamma B - \mu) - f(\frac{1}{2}mv^2 + \gamma B - \mu)}{f(\frac{1}{2}mv^2 - \gamma B - \mu) + f(\frac{1}{2}mv^2 + \gamma B - \mu)} = \frac{\frac{1}{1+e^{\beta(\frac{1}{2}mv^2-\mu-\gamma B)}} - \frac{1}{1+e^{\beta(\frac{1}{2}mv^2-\mu+\gamma B)}}}{\frac{1}{1+e^{\beta(\frac{1}{2}mv^2-\mu-\gamma B)}} + \frac{1}{1+e^{\beta(\frac{1}{2}mv^2-\mu+\gamma B)}}} \\
&\approx \frac{\frac{1}{1+e^{\beta(\frac{1}{2}mv^2-\mu)}(1-\beta\gamma B)} - \frac{1}{1+e^{\beta(\frac{1}{2}mv^2-\mu)}(1+\beta\gamma B)}}{\frac{1}{1+e^{\beta(\frac{1}{2}mv^2-\mu)}} + \frac{1}{1+e^{\beta(\frac{1}{2}mv^2-\mu)}}} \approx \frac{e^{\beta(\frac{1}{2}mv^2-\mu)}}{e^{\beta(\frac{1}{2}mv^2-\mu)}+1}\beta\gamma B \\
&= \frac{1}{1+e^{-\beta(\frac{1}{2}mv^2-\mu)}}\beta\gamma B + \mathcal{O}(B^3)
\end{aligned}$$

$\alpha(B;\mu)$ is an odd function of B, so there are only odd powers of B.
Within the approximation for low B ($\beta\gamma B \ll 1$), the maximal value of $\alpha(B;\mu)/B$ is $\beta\gamma$ and it is valid for $e^{-\beta(\frac{1}{2}mv^2-\mu)} \ll 1$ meaning we filter velocities at $v^2 \gg \frac{2}{m}(T+\mu)$