Ex 6030: Thermionic emission of electrons

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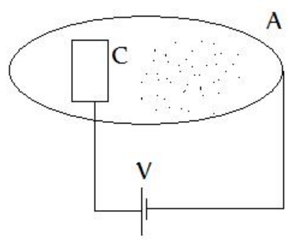
The problem:

A piece of metal ("Cathode") is placed inside a vacuum metal tube ("Anode"). The cathode has a work function W and surface A and is held at temperature T .

Calculate the saturation current I_s $(V \to \infty)$.

a)For $T \ll W$

b)For $T \gg W$



The solution:

a) Under the assumption that the electron gas inside the metal is in a state of quasi-static thermal equilibrium we use the velocity distribution:

$$F_{(v)} = 2 \cdot L^3 \cdot (\frac{m}{2\pi})^3 \cdot f(\frac{1}{2}mv^2 - \mu) \cdot d^3v \tag{1}$$

Now, understanding that only incident electrons with kinetic energy larger than the potential barrier will escape $(V_z \geq \sqrt{\frac{2\cdot(W+\mu_F)}{m}})$ and using the incident flux on a wall perpendicular to the Z axis , we get:

$$J = \int 2 \cdot \frac{F_{(v)}}{L^3} \cdot V_z = 2 \cdot (\frac{m}{2\pi})^3 \cdot \int_{V_{Z_{min}}}^{\infty} V_z dV_z \cdot \int_0^{\infty} \frac{2\pi V' dV'}{e^{-\beta(\frac{1}{2}mv_z^2 - \mu)} \cdot e^{\beta\frac{1}{2}mv'^2} + 1} \tag{2}$$

Integrating over the $x-y$ velocities and using a dimensionless variable $x = \beta W$:

$$J = 2 \cdot \frac{m^2}{(2\pi)^3} \cdot 2\pi \cdot T \cdot \int_{V_{Z_{min}}}^{\infty} ln(1 + e^{-\beta(\frac{1}{2}mv_z^2 - \mu)}) V_z dV_z = \frac{4\pi m}{(2\pi)^3} \cdot T^2 \cdot \int_{\beta W}^{\infty} ln(1 + e^{-x}) dx \tag{3}$$

For $T \ll W$ we use $ln(1 + e^{-x}) \approx e^{-x}$:

$$J = \frac{4\pi m T^2}{(2\pi)^3} \cdot e^{-\beta W} \tag{4}$$

The current density is : $e \cdot J$, and for the saturation current all emitted electrons make it to the Anode :

$$I_s = e \cdot J \cdot A = e \cdot A \cdot \frac{4\pi m T^2}{(2\pi)^3} \cdot e^{-\beta W} \tag{5}$$

b) For $T \gg W$, realizing we are now in the classical regime :

$$\mu = Tln(n \cdot \lambda^3) \tag{6}$$

Subsituting into (4) we get :

$$J = n \cdot (\frac{T}{2\pi m})^{\frac{1}{2}} \cdot e^{-\beta \frac{1}{2} m v_z^2} \tag{7}$$