Ex 5969

$$Z = \frac{1}{(N!)^2}\left(\frac{1}{\lambda_T^2}\right)^{2N} \cdot L^{4N}\int_0^1 d^{4N}\tilde{r}\; e^{\beta\sum_{\langle ij\rangle} q_i q_j \ln\left(\frac{L}{a}\tilde{r}_{ij}\right)}$$

$$\sum_{\langle ij\rangle} q_i q_j = q^2(N_{++}+N_{--}-N_{+-}) = -q^2 N$$

$$Z = \frac{1}{(N!)^2}\left(\frac{L}{\lambda_T}\right)^{4N}\left(\frac{L}{a}\right)^{-\beta q^2 N} \cdot \int d\tilde{r} \prod |r_i - r_j|^{\beta q_i q_j}$$

$$\Longrightarrow \quad Z = C\, L^{\alpha}, \qquad \alpha = (4-\beta q^2)N$$

C is implied by the above expression

$$P = \frac{1}{\beta}\frac{\partial \ln Z}{\partial A} = \frac{\alpha T}{2A} \qquad (A \equiv L^2)$$

$$(\alpha > 0) \quad \Rightarrow \quad T_c = q^2/4$$

$$F \approx 2T\ln(N!) + 4NT\ln\left(\frac{\lambda_T}{L}\right) + q^2 N \ln\left(\frac{L}{a}\right)$$

$$\mu = 2T\ln\left[N\cdot\left(\frac{\lambda_T}{L}\right)^2\right] + q^2\ln\left(\frac{L}{a}\right)$$

$$N = \left(\frac{L}{\lambda_T}\right)^2\left(\frac{a}{L}\right)^{\frac{2T_c}{T}} e^{\mu/2T}$$