

Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed
Department of Physics, Ben-Gurion University, Beer-Sheva 84105, Israel

This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

===== [Exercise 5968]

2D coulomb gas

N ions of positive charge q , and N ions with negative charge $-q$ are constrained to move in a two dimensional square of side L . The interaction energy of charge qi at position ri with another charge qj at rj is $-qiqj\ln|ri - rj|$ where $qi, qj = q$. Prove that $Z(\beta, L) = L^{N(4-\beta q^2)} f(\beta)$.

Estimate $f(\beta)$ for the case $N = 1$, and explain what happens if $\left(\frac{1}{\beta}\right) < \frac{q^2}{2}$. Discuss now the case $N \gg 1$. Explain what happens if $\left(\frac{1}{\beta}\right) < \frac{q^2}{4}$.

Hint: The partition function is in general a monotonic increasing function of the volume. It follows, for this particular model system, that $f(\beta) = \infty$.at low temperatures. The $N = 1$ case can be used in order to illuminate the reason for this divergence. Explain what is the additional ingredient that is required in order to stabilize the physics of this model.