

Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

===== [Exercise 5969]

2D Coulomb gas

N ions of positive charge q and N ions of negative charge $-q$ are constrained to move in a two dimensional square of side L and area $A = L^2$. The interaction energy of charge q_i at position r_i with another charge q_j at position r_j is $-q_i q_j \ln[|r_i - r_j|/a]$, where $q_i, q_j = \pm q$ and a is a microscopic length scale. The mass of the ions is m .

- (a) By rescaling space variables to $r_i := r_i/L$, the partition function can be written as $Z(L) = CL^\alpha$, where C does not depend on L . Find α . Hint: $\sum_{i,j} q_i q_j$ has a very simple dependence on N .
- (b) Calculate the pressure, and show that for $T < T_c$ the system is unstable. Determine what is T_c . Comment on the reason for this instability.
- (c) Determine what is C if the dimensionless dr integral is approximated by unity. In particular verify that your expression is strictly correct if $q = 0$.
- (d) Find the chemical potential $\mu(T, N, L)$, and solve for $N(\mu, T, L)$, using C of the previous item.