# Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed
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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

## ======= [Exercise 5963]

**Baruch's C20.**

Stoner ferromagnetism: The conduction electrons in a metal can be treated as a gas of fermions of spin $\frac{1}{2}$ (with up/down degeneracy), and density $n = N/V$ . The Coulomb repulsion favors wave functions which are antisymmetric in position coordinates, thus keeping the electrons apart. Because of the full (position and spin) antisymmetry of fermionic wave functions, this interaction may be approximated by an effective spin-spin coupling which favors states with parallel spins. In this simple approximation, the net effect is described by an interaction energy

$$U = \alpha \frac{N_+ N_-}{V}$$

where $N_+$ and $N_- = N - N_+$ are the numbers of electrons with up and down spins, and V is the volume.

(a) Define $n_\pm = N_\pm/V = n/2 \pm \delta$ and assume $\delta \ll n$. Expand the total energy at temperature $T = 0$ (kinetic and interaction) to 4th order in $\delta$ and find the critical value $\alpha_c$ such that for $\alpha > \alpha_c$ the electron gas can lower its total energy by spontaneously developing a magnetization. (This is known as the Stoner instability.)

(b) Explain the instability qualitatively, and sketch the behavior of the spontaneous magnetization as a function of $\alpha$.

(c) Reconsider (a) at finite but low temperatures $T$, and find $\alpha_c(T)$ (consider the effect to the lowest nonzero order of $T$).