

Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

===== [Exercise 5963]

Stoner ferromagnetism

Consider Fermi gas of N spin $1/2$ electrons, at temperature $T = 0$. Define N_+ and N_- as the number of “up” and “down” electrons respectively, such that $N = N_+ + N_-$. Due to the antisymmetry of the total wave function the energy of the system is $U = \alpha N_+ N_- / V$, where V is the volume. Note that this interaction favors parallel spin states. Define the magnetization as $M = (N_+ - N_-) / V$.

- (a) Write the total energy $E(M)$, including both the kinetic energy and the interaction, and expand up to 4th order in M .
- (b) Find the critical value α_c , such that for $\alpha > \alpha_c$ the electron gas can lower its total energy by spontaneously developing magnetization. This is known as the Stoner instability.
- (c) Explain the instability qualitatively, and sketch the behavior of the spontaneous magnetization versus α .
- (d) Repeat (a) at finite but low temperatures T , and find $\alpha_c(T)$ to second order in T .

Guidance: In the last item explain why the energy $E(M)$ should be replaced by the M -constrained “free energy” $F(M)$. Use known results [Patria] for the free energy of electrons at finite temperature.