5955: Change of boiling point with altitude

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The problem:

Consider an atmosphere as an ideal gas whose average mass is $m_A = 30$ gr/mole, atmospheric pressure at sea level ($h = 0$) equals P_0 and is at uniform temperature of $T_A = 27\,°C$.
We take liquid whose latent heat Q is 1000 cal/mole and we find that its boiling point is $95\,°C$ at the top of a mountain and $105\,°C$ at the bottom (sea level).
Asume that the gas phase of the liquid is an ideal gas with density much lower than that of the liquid.

(1) Calculate the atmospheric pressure P_A as a function of height h.

(2) Calculate the liquid pressure as a function of its temerature.

(3) From above deduce the height of the mountain.

The solution:

(1)
The probability to find a particle at height h is

$$p(h) \propto e^{-\beta mgh}$$

the number of particles at height h is

$$N_A(H) = N_A(0)e^{-\beta mgh}$$

The gas is an ideal gas at uniform temperature and therefore

$$P_A(h) = P_A(0)e^{-\beta mgh}$$

(2)
We know that F_H is an analitical function and $dF_H = -SdT - PdV$. From the analitical property of the F_H function

$$\frac{dP}{dT} = \frac{\partial^2 F_H}{\partial T \partial V} = \frac{\partial^2 F_H}{\partial V \partial T} = \frac{dS}{dV} = \frac{TdS}{TdV} \approx \frac{Q}{TV_{gas}} = \frac{Q}{k_b T^2}P$$

therefore

$$\frac{dP}{P} = -\frac{Q}{k_b T^2}dT$$

$$\Rightarrow Ln(P(T_h)) - Ln(P(T_{h=0})) = \frac{Q}{k_b}(\frac{1}{T_{h=0}} - \frac{1}{T_h}) \Leftrightarrow P_g(T) = P_g(T_{h=0})e^{\frac{Q}{k_b}(\frac{1}{T_{h=0}} - \frac{1}{T_h})}$$

(3)
We define $T_{h=0} = T_0$
At steady state the pressure of the atmosphere and the gas must be equal

$$P_g(T_h) = P_A(h) \Leftrightarrow P_0 e^{\frac{Q}{k_b}(\frac{1}{T_0} - \frac{1}{T_h})} = P_0 e^{-\frac{mg}{k_b T_A}z} \Leftrightarrow Q(\frac{1}{T_0} - \frac{1}{T_h}) = -\frac{m_A g}{T_A}h$$

We know that $1cal/mol = 4.184J/mol$ and therefore:

$$h = -4184 * (\frac{1}{378} - \frac{1}{368}) * 300/0.3 = 300.8m$$

Remark:
In case of water the latent heat is $\approx 40KJ/mole$ and we get the same way that

$$h \approx -40000 * (\frac{1}{378} - \frac{1}{368}) * 300/0.3 = 2875m$$

This fit with the fact that the boiling point of water decreases by 1^0C for increase of 286 meters in height.