

Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

===== [Exercise 5841]

Disorder averaging

Consider a system with random impurities. An experiment measures one realization of the impurity distribution and many experiments yield an average denoted by $\langle \dots \rangle$. Consider the free energy as being a sum over N independent subsystems, i.e. parts of the original system, with average value $F = (1/N) \sum_{i=1}^N F_i$; the subsystems are identical in average, i.e. $\langle F_i \rangle = \langle F \rangle$.

- (a) The subsystems are independent, i.e. $\langle F_i F_j \rangle = \langle F_i \rangle \langle F_j \rangle$ for $i \neq j$, although they may interact through their surface. Explain this.
- (b) Show that $\langle (F - \langle F \rangle)^2 \rangle \sim 1/N$ so that even if the variance $\langle (F_i - \langle F \rangle)^2 \rangle$ may not be small any measurement of F is typically near its average.
- (c) Would the conclusion (b) apply to the average of the partition function Z , i.e. replacing F_i by Z_i ?