Long range Ising:

a. $E_0 = -b \sum_{n \neq n'}' 1/(n-n')^\gamma$ count each pair once

For each pair with $n \leq 0$ and $n' \geq 0$ interaction is reversed

$$E_{wall} = 2b \sum_{n=-\infty}^{-1} \sum_{n'=0}^{\infty} \frac{1}{|n-n'|^\gamma} \simeq 2b \int_{-\infty}^{0} dn \int_0^\infty dn' \frac{1}{(n'-n)^\gamma}$$

Convergence at $n \to \infty$ is needed, hence $\sum \to \int dn$ is valid

$$= 2b \int_{-\infty}^{0} dn \cdot \frac{-1}{\gamma-1} \frac{1}{(n'-n)^{\gamma-1}} \Big]_{n'=0}^{\infty}$$ if $\gamma < 1$ divergence already here.

if $\gamma > 1$:

$$= \frac{2b}{\gamma-1} \int_{-\infty}^{0} dn \frac{1}{(-n)^{\gamma-1}} = \frac{2b}{\gamma-1} \int_0^\infty dn \frac{1}{n^{\gamma-1}} = \frac{2b}{(\gamma-1)(\gamma-2)} \frac{-1}{n^{\gamma-2}} \Big]_0^\infty$$

if $\gamma < 2$ $\quad E_{wall} \to \infty > k_B T \ln N$ entropy of wall.
$\sim N^{2-\gamma}$

b. $Z = \sum_{\sigma_n = \pm} e^{+\beta J \sum \sigma_n \sigma_{n+1}}$ $=$ change $\sigma_n \to -\sigma_n$ on odd sites

$$= \sum e^{-\beta J \sum \sigma_n \sigma_{n+1}}$$

Bi-partite lattices (in any dimension) satisfy $F(J) = F(-J)$

i.e. having two sublattices, spin i has neighbors only on the other sublattice. Changing $\sigma_i \to -\sigma_i$ only on <u>one</u> sublattice shows $F(J) = F(-J)$