Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

[Exercise 5966]

Baruch's C23.

(a) Consider the Ising spin model on a bipartite lattice, i.e. it has two sublattices A,B such that each site on lattice A has its nearest neighbors on sublattice B, and vice versa. The Hamiltonian is

$$\mathcal{H} = -J \sum_{\mathbf{n},\boldsymbol{\delta}} \sigma(\mathbf{n})\sigma(\mathbf{n}+\boldsymbol{\delta}) - h \sum_{\mathbf{n}} \sigma(\mathbf{n}) \tag{1}$$

where $\mathbf{n}$ are the lattice sites, $\boldsymbol{\delta}$ labels the nearest neighbors, h is proportional to a magnetic field and $\sigma(\mathbf{n}) = \pm$. For $h = 0$ show that the free energy satisfies $F(J,T) = F(-J,T)$, hence the critical temperatures satisfy $T_c^F = T_c^{AF}$ for the ferromagnetic ($J > 0$) and anti-ferromagnetic ($J < 0$) transitions. Define the order parameters at $T < T_c$ and the magnetic susceptibilities to h at $T > T_c$ and find their relationship, if any.

(b) Consider the one dimensional Ising model with the Hamiltonian $\mathcal{H} = -\sum_{n,n'} J(n-n')\sigma(n)\sigma(n')$ with $\sigma(n) = \pm 1$ at each site n and $J(n) = b/n^\gamma$ is a long range interaction and $b > 0$. Find the energy of a domain wall (i.e. $n < 0$ spins are $-$ and $n \geq 0$ are $+$) and show that the argument for the absence of spontaneous magnetization at finite temperatures fails when $\gamma < 2$.