Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

===== [**Exercise 5810**]
Mechanical model for symmetry breaking

The following mechanical model illustrates the symmetry breaking aspect of second order phase transitions. An air tight piston of mass M is inside a tube of cross sectional area a. The tube is bent into a semicircular shape of radius R. On each side of the piston there is an ideal gas of N atoms at a temperature T. The volume to the right of the piston is $aR\left(\frac{\pi}{2} - \varphi\right)$ while to the left is $aR\left(\frac{\pi}{2} + \varphi\right)$. The free energy of the system has the form
$F(T, \varphi) = MgR\cos\varphi - NT\left[2 + \ln\left(aR\left(\frac{\pi}{2} - \varphi\right)/N\lambda^3\right) + \ln\left(aR\left(\frac{\pi}{2} + \varphi\right)/N\lambda^3\right)\right]$.
Explain the terms in F. Interpret the minimum condition for $F(\varphi)$ in terms of the pressures in the two chambers. Expand F up to 4th order in φ. Show that there is a symmetry breaking transition and find the critical temperature T_c. Describe what happens to the phase transition if the number of atoms on the left and right of the piston is $N(1 + \delta)$ and $N(1 - \delta)$ respectively. Note that it is sufficient to consider $|\delta| << 1$ and include a term $\varphi\delta$ in the expansion. At a certain temperature the left chamber (containing $N(1 + \delta)$ atoms) is found to contain a droplet of liquid coing with its vapor. Which of the following statements may be true at equilibrium:

(a) The right chamber contains a liquid coming with its vapor;

(b) The right chamber contains only vapor;

(c) The right chamber contains only liquid.