Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

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Correlation function for ferromagnet - mean field

Consider a ferromagnet with magnetic moments $m(\mathbf{r})$ on a simple cubic lattice interacting with their nearest neighbors. [The symmetry is an Ising type, i.e. $m(\mathbf{r})$ is the moment's amplitude in a preferred direction]. The ferromagnetic coupling is J and the lattice constant is a. Extend the mean field theory to the situation that the magnetization is not uniform but is slowly varying:

(a) Find the mean field equation in terms of $m(\mathbf{r})$, its gradients (to lowest order) and an external magnetic $H(\mathbf{r})$, which in general can be a function of $\mathbf{r}$.

(b) Consider $T > T_c$ where T_c is the critical temperature so that only lowest order in $m(\mathbf{r})$ is needed. For a small $H(\mathbf{r})$ find the response $m(\mathbf{r})$ and evaluate it explicitly in two limits: (i) uniform H, i.e. find the susceptibility, and (ii) $H(\mathbf{r}) \sim \delta^3(\mathbf{r})$. Explain why in case (ii) the response is the correlation function and identify the correlation length.