

Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed
Department of Physics, Ben-Gurion University, Beer-Sheva 84105, Israel

This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

===== [Exercise 5958]

Baruch's C04.

Model of ferroelectricity: Consider electric dipoles \mathbf{p} on sites of a simple cubic lattice which point along one of the crystal axes, $\pm\langle 100 \rangle$. The interaction between dipoles is

$$U = \frac{\mathbf{p}_1 \cdot \mathbf{p}_2 - 3(\mathbf{p}_1 \cdot \mathbf{r})(\mathbf{p}_2 \cdot \mathbf{r})/r^2}{4\pi\epsilon_0 r^3}$$

where \mathbf{r} is the distance between the dipoles, $r = |\mathbf{r}|$ and ϵ_0 is the dielectric constant.

- (a) Assume nearest neighbor interactions and find the ground state configuration. Consider either ferroelectric (parallel dipoles) or anti-ferroelectric alignment (anti-parallel) between neighbors in various directions.
- (b) Develop a mean field theory for the ordering in (a) for the average polarization P at a given site at temperature T : Write a mean field equation for $P(T)$ and find the critical temperature T_c .
- (c) Find the susceptibility $\chi = \left(\frac{\partial P}{\partial E}\right)_{E=0}$ at $T > T_c$ for an electric field $E \parallel \langle 100 \rangle$, using the mean field theory.