

Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

===== [Exercise 5732]

Mean field for ferroelectricity

Consider electric dipoles \mathbf{p} that are situated on sites of a simple cubic lattice, which point along the crystal axes $\pm\langle 100 \rangle$. The interaction between dipoles is

$$U = \frac{\mathbf{p}_1 \cdot \mathbf{p}_2 - 3(\mathbf{p}_1 \cdot \mathbf{r})(\mathbf{p}_2 \cdot \mathbf{r})/r^2}{4\pi r^3}$$

where \mathbf{r} is the distance between the dipoles, and $r = |\mathbf{r}|$.

- (a) Assume nearest neighbour interactions and find the ground state configuration. Consider either ferroelectric (parallel dipoles) or anti-ferroelectric alignment (anti-parallel) between neighbours in various directions.
- (b) Develop a mean field theory for the ordering in (a) for the average polarization P at temperature T . Write the mean field equation for $P(T)$, and find the critical temperature T_c .
- (c) Within the mean field approximation find the susceptibility $\chi = (\partial P / \partial E)_{E=0}$ for $T > T_c$ with respect to the electric field $E||\langle 100 \rangle$.