Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

[Exercise 5721]

Mean field for antiferromagnetism

Consider Ising model on a 2D lattice with antiferromagnetic interaction ($\epsilon = -\epsilon_0$). You can regard the lattice as composed of two sublattices A and B, such that $M = \frac{1}{2}(M_A + M_B)$ is the averaged magnetization per spin, and $M_s = \frac{1}{2}(M_A - M_B)$ is the staggered magnetization

(a) Explain the claim: for zero field ($h = 0$), Ising antiferromagnet is the *same* as Ising ferromagnet, where M_s is the order parameter. Write the expression for $M_s(T)$ for $T \sim T_c$, based on the familiar solution of the ferromagnetic case.

(b) Given h and ϵ_0, find the coupled mean-field equations for M_A and M_B.

(c) Find the critical temperature T_c for $h = 0$, and also for small h. Hints: for $h = 0$ use the same procedure of expanding $\text{arctanh}(x)$ as in the ferromagnetic case; for small h you may use the most extreme simplification that does not give a trivial solution.

(d) Find the critical magnetic field h_c above which the system no longer acts as an antiferromagnet at zero temperature.

(e) Find an expression for the susceptibility $\chi(T)$, expressed as a function of the staggered magnetization $M_s(T)$.

(f) In the region of $T \sim T_c$ give a linear approximation for $1/\chi$ as a function of the temperature T