Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

[Exercise 5956]

Baruch's C02.

Apply the mean field approximation to the classical spin-vector model

$$\mathcal{H} = -J\sum_{\langle i,j\rangle} \mathbf{s}_i \cdot \mathbf{s}_j - \mathbf{h}\cdot\sum_i \mathbf{s}_i$$

where $\mathbf{s}_i$ is a unit vector and i, j are neighboring sites on a lattice; each pair is counted once. The lattice has N sites and each site has γ neighbors.

(a) Define a mean field $\mathbf{h}^{eff}$ and evaluate the partition function Z in terms of $\mathbf{h}^{eff}$.

(b) Find an equation for the magnetization $M(T) = \langle\cos\theta_i\rangle$ at $\mathbf{h} = 0$ where θ_i is the angle relative to the orientation of $\mathbf{M}$. Find the transition temperature T_c.

(c) The mean field hamiltonian (at $\mathbf{h} = 0$) is $\mathcal{H}^{MF} = -J\gamma M\sum_i s_i + \frac{1}{2}J\gamma M^2 N$. Explain this form, derive the free energy F, and re-derive the equation for $M(T)$ in (b) by a minimum condition.

(d) Identify exponents γ, β as $T \to T_c$ for the susceptibility $\chi \sim (T - T_c)^{-\gamma}$ above T_c and for $M \sim (T_c - T)^{\beta}$ below T_c.

(e) Show that there is a jump in C_V at T_c.