Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

======= [Exercise 5713]

Mean field approximation for a classical Heisenberg model

Apply the mean field approximation to the classical spin vector model

$$\mathcal{H} = -\epsilon \sum_{\langle i,j \rangle} \boldsymbol{s}_i \cdot \boldsymbol{s}_j - \boldsymbol{h} \cdot \sum_i \boldsymbol{s}_i$$

where $\boldsymbol{s}_i$ is a unit vector and i, j are neighboring sites on a lattice with coordination number c. The lattice has N sites and each site has c neighbors.

(a) Assume that $\boldsymbol{h} = (0, 0, h)$, define a mean field $\boldsymbol{h}_{eff}$, and evaluate the partition function Z in terms of $\boldsymbol{h}_{eff}$.

(b) Define θ_i as the inclination angle of $\boldsymbol{s}_i$ with respect to $\boldsymbol{h}$. Assume that at equilibrium $\boldsymbol{s}_i = (0, 0, M)$, where $M = \langle \cos\theta \rangle$. Find the equation for M, and find the transition temperature T_c.

(c) Write an expression for the mean field energy of the system assuming that $M(T)$ is known.

(d) Identify exponents γ and β that describe the susceptibility $\chi \sim (T - T_c)^{-\gamma}$ above T_c, and the magnetization $M \sim (T_c - T)^{\beta}$ below T_c.

(e) Find the jump in the heat capacity C_V at T_c.