

## Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed  
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This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

### ===== [Exercise 5660]

#### Ising spins mediated by adsorption sites

Consider a ring along which  $M$  absorption sites are arranged. The number of particles that can be absorbed at site  $i$  is  $n_i = 0, 1$ . Between every two absorption sites a spin  $\sigma_i = \pm 1$  is located. The ring is surrounded by gas in temperature  $T$  and chemical potential  $\mu$ . The absorption energy is  $\epsilon > 0$  if the two adjacent spins are in the same direction, and  $-\epsilon$  otherwise.

1. Write an expression for the energy  $E[\sigma_i, n_i]$  of a given configuration.
2. Calculate the partition function  $\mathcal{Z}(\beta, \mu)$  using the transfer matrix method. Write what is  $T_{\sigma_i, \sigma_{i+1}}$  in this problem.
3. Find the Helmholtz function  $F(T, \mu)$  assuming  $M \gg 1$ .
4. Write an expression for the average number of adsorbed particles  $N = \sum_i \langle n_i \rangle$  as a function of  $(\beta, \mu)$ .
5. Write an expression for the correlation length  $\xi$  that characterizes arrangement of the spins in the system.