

Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

===== [Exercise 5651]

Baruch's C24.

Consider a one dimensional Ising model of spins $\sigma_i = \pm 1$, $i = 1, 2, 3, \dots, N$ and $\sigma_{N+1} = \sigma_1$. Between each two spins there is a site for an additional atom, which if present changes the coupling J to $J(1 - \lambda)$. The Hamiltonian is then

$$H = -J \sum_{i=1}^N \sigma_i \sigma_{i+1} (1 - \lambda n_i)$$

where $n_i = 0$ or 1 and there are $N' = \sum_{i=1}^N n_i$ atoms ($N' < N$), i.e. on average $\langle n_i \rangle = N'/N$.

- (a) Evaluate the partition sum by allowing all configurations of spins and of atoms.
- (b) If the atoms are stationary impurities one needs to evaluate the free energy F for some random configuration of the atoms and then average F over all configurations. (The reasons for this average are given in Ex. C25). Evaluate the average F . Find the entropy difference of (a) and (b) and explain its origin.