

## Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed  
*Department of Physics, Ben-Gurion University, Beer-Sheva 84105, Israel*

This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

### ===== [Exercise 5650]

#### Ising spins with interaction that is mediated by atoms

Consider a one dimensional Ising model of spins  $\sigma_i = \pm 1$ , where  $i = 1, 2, 3, \dots, N$  and  $\sigma_{N+1} = \sigma_1$ . Between each two spins there is a site for an additional atom, which if present changes the coupling  $J$  to  $J(1 - \lambda)$ . The Hamiltonian is then  $H = -J\sum_i \sigma_i \sigma_{i+1} + 1(1 - \lambda n_i)$ , where  $n_i = 0$  or  $1$ . There are  $N' = \sum n_i$  atoms, so that  $N' < N$ . Evaluate the partition sum by allowing all configurations of spins and of atoms. If the atoms are stationary impurities one needs to evaluate the free energy  $F$  for some given random configuration of the atoms: Then one can average  $F$  over all configurations. Evaluate the averaged  $F$ . Find the entropy difference between the two results and explain its origin.