Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed
Department of Physics, Ben-Gurion University, Beer-Sheva 84105, Israel

This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

═══ [Exercise 5651]

Ising spins with interaction that is mediated by atoms

Consider a one dimensional Ising model of spins $\sigma_i = \pm 1$ labeled $i = 1, 2, 3, ..., M$, with periodic boundary condition. Between each two spins there is a site $n_i = 0, 1$ that can be occupied by an atom. If the atom is present the feromagnetic coupling is decreased from J to $(1 - \lambda)J$.

(1) Evaluate the partition sum assuming that there are N atoms in the M sites. Allow all configurations of spins and of atoms. Calculate the free energy F.

(2) If the atoms are stationary impurities one needs to evaluate the free energy F for some random configuration of the atoms. What is the entropy difference between the results?