Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

======= [Exercise 5971]

Baruch's C28.

Potts model in 1-dimension (1d) . A set of N atoms, each with p states is arranged on a 1d chain with periodic boundary conditions. The atom at the n-th site is in a state i_n that is chosen from the set $\{1, 2, ..., p\}$. Two neighboring atoms at sites n and $n+1$, respectively, have an interaction energy $-J$ ($J > 0$) if they are in the same state, i.e. $i_n = i_{n+1}$, and 0 interaction otherwise. The Hamiltonian is therefore

$$\mathcal{H} = -J \sum_{i=1}^{N} \delta_{i_n, i_{n+1}}$$

where $\delta_{i_n, i_{n+1}}$ is the Kronecker symbol, and the boundary conditions are $i_{N+1} = i_1$.

(a) Derive the free energy for $p = 2$. (Consider here and below the limit $N \to \infty$.)

(b) Derive the free energy for a general p. Hint: Show that the eigenvector of the transfer matrix whose all entries are equal has the largest eigenvalue.

(c) Find the internal energy E at the low and high temperature limits and interpret the results.