

Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed
Department of Physics, Ben-Gurion University, Beer-Sheva 84105, Israel

This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

===== [Exercise 5645]

Potts model in one dimension

A set of N atoms is arranged on a one-dimensional chain. Each atom has p possible *orientations*, labelled by $\sigma = 1, 2, \dots, p$. Two neighboring atoms σ_i and σ_j have a negative interaction energy $-\varepsilon$ if they are in the same orientation, and zero otherwise. It is useful to define bond variables $s_i = \sigma_{i+1} - \sigma_i \pmod{p}$.

- (1) The partition function $Z_{\text{chain}}(\beta)$ of an open chain can be written as $Z = Aq^{N-1}$. Write what are A and q . Tip: the partition sum factorizes in the “bond” representation.
- (2) The partition function $Z_{\text{ring}}(\beta)$ of a closed chain, with periodic boundary conditions, can be written as $Z = \text{trace}(T^N)$. Write what is the matrix T for $p = 4$.
- (3) Find what are the eigenvalues of the transfer matrix T for general p , and deduce an explicit expression for $Z_{\text{ring}}(\beta)$. Tip: The T matrix is diagonal in the “momentum” representation.
- (4) Find the energy per atom $E(T)/N$ at the $N \rightarrow \infty$ limit. Express your results in terms of (p, ε, T) .