HW 2008 8.3 | (HW 2009 8.2)

C21

Ising + Long range interaction , $J>0$, $h=\mu_B H$

$$\mathcal{H} = -\frac{J}{2N}\sum_{\forall i,j} S_i S_j - h\sum_i S_i$$

א) נגדיר $m = \frac{1}{N}\sum_i S_i$ (נקבל)

$$\mathcal{H} = -\frac{1}{2}JNm^2 - hNm$$

מכיוון שהסכום על כל ij דרוש $\frac{1}{N}$ למנוע התבדרות חזקה

של האנרגיה בגבול התרמודינמי, משמע $J \to \frac{J}{N}$

כדי ש- $\mathcal{H}(N) \sim N$ = extensive

ולא $\sim N^2$

ב) נשים לב שלכל קונפיגורציה, האנטרופיה

$$S = k_B \ln\left(\frac{N!}{N_+!\,N_-!}\right) \qquad N_+ = \frac{N}{2}(1+m)\,,\quad N_- = \frac{N}{2}(1-m)$$

נזכור ש- $U = \langle \mathcal{H}\rangle$ $\qquad U = -\frac{1}{2}JN\langle m^2\rangle - hN\langle m\rangle$

בגבול קירוב MF רק ש- $\langle m^2\rangle \approx \langle m\rangle^2$ (מזניחים פלקטואציות)

מכאן, (נסמן $m=\langle m\rangle$ כדי לא לסחוב $\langle\rangle$) $\qquad F_0 = U - TS$

$$= -\frac{1}{2}NqJm^2 - Nmh - kT\left(N\ln N - \frac{N}{2}(1+m)\ln\left[\frac{N}{2}(1+m)\right] - \frac{N}{2}(1-m)\ln\left[\frac{N}{2}(1-m)\right]\right)$$

$$F_0(m=0) = -kT\left(N\ln N - N\ln\frac{N}{2}\right) = -kT N\ln 2$$

$$\frac{\partial F_0}{\partial m} = -NqJm - Nh - kT\left[-\frac{N}{2}\ln\left(\frac{N}{2}(1+m)\right) + \frac{N}{2}\ln\left(\frac{N}{2}(1-m)\right)\right] = 0$$

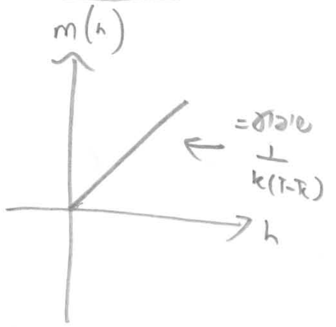
$$\frac{qJm+h}{kT} = \frac{1}{2}\ln\frac{1-m}{1+m} = \operatorname{Arctanh}(m)$$

$$m = \tanh\left(\frac{h+qJm}{kT}\right) \rightarrow \boxed{k_B T_c = J}$$

נצייר

$$m(h) = \tanh\left(\frac{Jm+h}{kT}\right)$$

$\boxed{T>T_c}$

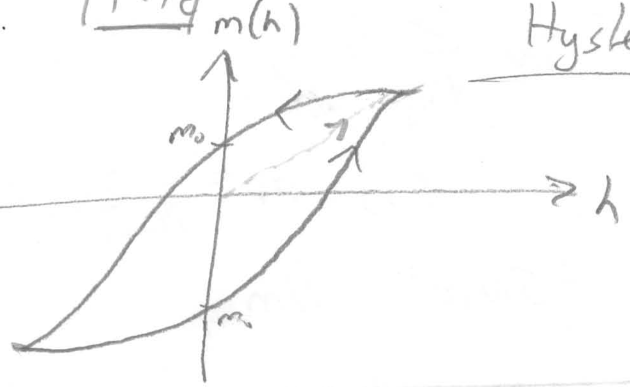


$$m(h) \approx \frac{Jm(h)+h}{kT}$$

$$m(h) = \frac{h}{kT-J} = \frac{h}{k(T-T_c)}$$

$\boxed{T<T_c}$

Hysteresis Loop



נחקור $f_0(m)$, כאשר $\frac{\partial f_0}{\partial m}=0$,

$$\left.\frac{\partial^2 f_0}{\partial m^2}\right|_{m=0} = -NqJ + kT\frac{N}{2}\left(\frac{1}{1+m}+\frac{1}{1-m}\right)\Big|_{m=0} = -NqJ + kTN$$

$$= kN(T-T_c)$$

$$\left.\frac{\partial^3 F}{\partial m^3}\right|_{m=0} = kT\frac{N}{2}\left(\frac{-1}{(1+m)^2}+\frac{1}{(1-m)^2}\right) = 0$$

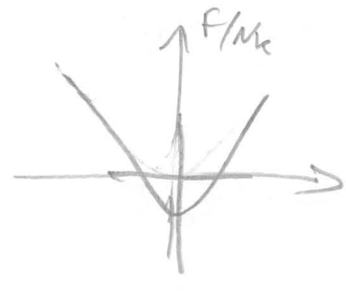
$$\left.\frac{\partial^4 F}{\partial m^4}\right|_{m=0} = kT\frac{N}{2}(2+2) = 2kTN$$

אמון

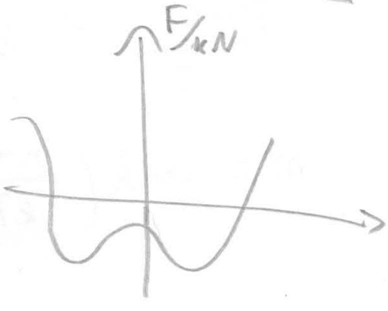
$$\frac{F}{kN} = \frac{1}{2}(T-T_c)m^2 + \frac{1}{4!}\cdot 2Tm^4 - T\ln 2$$

כעת זכור ש-

$T>T_c$



$T<T_c$



hysteresis [illegible]

$$m = \tanh\left(\frac{mT_c}{T}+\beta h\right)$$

$$\frac{\partial m}{\partial h} = \frac{1}{\cosh^2\left(\frac{mT_c}{T}+\beta h\right)}\cdot\left[\frac{T_c}{T}\frac{\partial m}{\partial h}+\beta\right]$$

$$\frac{\partial m}{\partial h}\left[1-\frac{T_c/T}{\cosh^2\left(\frac{mT_c}{T}+\beta h\right)}\right] = \frac{\beta}{\cosh^2(\cdot)}$$

$$\boxed{\frac{\partial m}{\partial h} = \frac{\beta}{\cosh^2\left(\frac{mT_c}{T}+\beta h\right)-T_c/T}}$$

$$\left.\frac{\partial m}{\partial h}\right|_{h=0} = \frac{\beta}{\cosh^2\left(\frac{mT_c}{T}\right)-T_c/T} \quad \left[\begin{array}{ll} >0 & T \ll T_c \\ <0 & T \lesssim T_c \end{array}\right.$$

$$\left.\cosh^2(x)\right|_{x\ll 1} \approx 1+x^2 \rightarrow \left.\cosh^2\left(\frac{mT_c}{T}\right)\right|_{T\lesssim T_c} \approx 1+m^2\left(\frac{T_c}{T}\right)^2$$

but $\left.m\right|_{h=0} \approx m\frac{T_c}{T}-\frac{1}{3}m^3\left(\frac{T_c}{T}\right)^3$

$$\rightarrow m = \pm\sqrt{\frac{3\left(1-\frac{T}{T_c}\right)}{(T_c/T)^2}} = \pm\frac{T}{T_c}\sqrt{3\left(\frac{T_c-T}{T_c}\right)}$$