Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

======= [Exercise 5641]

Baruch's C21.

Consider the Ising model of magnetism with long range interaction: the energy of a spin configuration $\{s_i\}$ with $s_i = \pm 1$ on an arbitrary lattice is given by,

$$E = -\left(J/2N\right)\sum_{i,j} s_i s_j - h\sum_i s_i$$

where $J > 0$ and the sum is on all i and j (in the usual Ising model the sum is restricted to nearest neighbors) and $h = \mu_B H, H$ is the magnetic field.

(a) Write E in terms of $m = \sum_i s_i/N$ i.e. $E\left(m,h\right) = -\left(1/2\right)JNm^2 - hNm$; why is N included in the definition of the coupling J/N?

(b) Evaluate the free energy $F_0\left(m;T,h\right)$ assuming that it is dominated by a single m which is then a variational parameter. From the minima of F_0 find $m\left(h,T\right)$ and a ciritical temperature T_c. Plot qualitatively $m\left(h\right)$ above and below the transition.

(c) Plot qualitatively $F_0\left(m\right)$ for $T > T_c$ and $T < T_c$ with both $h = 0$ and $h \neq 0$. Explain the meanings of the various extrema.

(d) Expand $F_0\left(m;T,h=0\right)$ up to order m^4. What is the meaning of the m^2 coefficient?