Ex 5442 - coupled rotors

$$H = \frac{p^2}{2ma^2} - h\cos(\varphi)$$

$\chi = \frac{1}{2T}$ classical

$\chi = 2ma^2$ quantum $T=0$

$$R(x) = \frac{I_0'}{I_0}(2x) \quad , \quad I_0(x) \equiv \frac{1}{2\pi}\int_0^{2\pi} e^{x\cos(\varphi)}\,d\varphi$$

mean field $(c=2)$ $\tilde{h} = h + c\varepsilon M$

symetry breaking if $T < \frac{c\varepsilon}{2}$, $ma^2 > \frac{1}{2c\varepsilon}$

The exact calculation of χ implies $T_c = 0$

$$C(r) = \langle \cos\varphi_0 \cos(\varphi_0 + \phi_1 + \phi_2 + \cdots \phi_r) \rangle$$

$$= \frac{1}{2}\langle \cos\phi \rangle^r \equiv \frac{1}{2}\left[R\left(\frac{1}{2}\frac{\varepsilon}{T}\right)\right]^r \equiv e^{-r/\xi}$$

$$\chi = \frac{1}{T}\sum C(r) = \frac{1}{T}\coth\left(\frac{1}{2\xi}\right)$$