Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

======= [Exercise 2345]

Baruch's A26.

Polymer in two dimensions: Configurations of a polymer are described by a set of vectors $\mathbf{t}_i$ of length a in two dimensions (for i = 1,...,N), or alternatively by the angles ϕ_i between successive vectors, as indicated in the figure below. The energy of a configuration $\{\phi_i\}$ is

$$\mathcal{H} = -\kappa \sum_{i=1}^{N-1} \mathbf{t}_i \cdot \mathbf{t}_{\mathbf{i+1}} = -\kappa a^2 \sum_{i=1}^{N-1} \cos\phi_i$$

(a) Show that $\langle \mathbf{t}_n \cdot \mathbf{t}_m \rangle = a^2 \mathrm{e}^{-|n-m|/\xi}$ and obtain an expression for the "persistence length" $a\xi$; you can leave the answer in terms of simple integrals.

Hint: Show $\mathbf{t}_n \cdot \mathbf{t}_m = a^2 \mathrm{Re}\,\{\mathrm{e}^{i\sum_{j=n}^{m-1}\phi_j}\}$.

(b) Consider the end-to-end distance $\mathbf{R}$ as illustrated in the figure. Show that for $N \gg 1$, $\langle R^2 \rangle = a^2 N \coth(1/2\xi)$. [Note: $\sum_{j=1}^{n} x^j = x\frac{1-x^n}{1-x}$]