

Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed
Department of Physics, Ben-Gurion University, Beer-Sheva 84105, Israel

This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

===== [Exercise 5420]

Correlation function for Ising model

Consider the Ising model in one dimension with periodic boundary condition and with zero external field.

- (a) Consider an Ising spin σ_i ($\sigma_i = \pm 1$) at site i and explain why do you expect $\langle \sigma_i \rangle = 0$ at any temperature $T \neq 0$. Evaluate $\langle \sigma_i \rangle$ by using the transfer matrix method. What is $\langle \sigma_i \rangle$ at $T = 0$?
- (b) Find the correlation function $G(r) = \langle \sigma_1 \sigma_{r+1} \rangle$ and show that when $N \rightarrow \infty$ (N is the number of spins) $G(r)$ has the form $G(r) \sim e^{-r/\xi}$. At what temperature ξ diverges and what is its significance?