Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

======= [Exercise 5023]

Baruch's A22.

Consider a one-dimensional classical gas of N particles in a length L at temperature T. The particles have mass m and interact via a 2-body "hard sphere" interaction (x_i is the position of the i-th particle):

$$\begin{aligned} V(x_i - x_j) &= \infty \qquad |x_i - x_j| < a \\ &= 0 \qquad |x_i - x_j| > a \end{aligned}$$

(a) Evaluate the exact free energy F(T,L,N).

(b) Find the equation of state and identify the first virial coefficient; compare with its direct definition.

(c) Show that the energy is $E = Nk_BT/2$. Why is there no effect of the interactions on E ?

(d) In three dimensions $V(|\mathbf{r}_i - \mathbf{r}_j|)$ is defined as above with $\mathbf{r}$ the position vector. Explain why should the effective volume satisfy $V > V_{eff} > V - v_0 N$ where v_0 is the excluded volume for each particle (i.e. $4\pi a^3/3$, where a is the particle's diameter) and $Z = V_{eff}^N/(\lambda^{3N} N!)$. Find $E(T)$ and explain how can it be consistent with the presence of a phase transition.