Exercises in Statistical Mechanics

This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

===== [Exercise 5011]

Hard spheres in a box

Consider a gas of N hard spheres in a box. A single sphere occupies a volume ω , while its center of mass can explore a volume V (if the rest of the space is empty). There are no other interactions between the spheres except for the constraints of hard core exclusion.

(a) Calculate the partition function for this gas. You will need to integrate over the spatial part of phase space. Use the approximation

$$\int_{|x_i-x_j|>a} d^3x_1 d^3x_2...d^3x_N \approx V(V - \omega)(V - 2\omega)...(V - (N - 1)\omega)$$

and explain under which physical circumstances this approximation is valid.

(b) Calculate the entropy. Use the approximation

$$(V - a\omega)(V - (N - a)\omega) \approx (V - N\omega/2)^2$$

where a is an integer. When is this approximation valid?

(c) Find the equation of state.

(d) Show that the isothermal compressibility $\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$ is always positive.