Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed
Department of Physics, Ben-Gurion University, Beer-Sheva 84105, Israel

This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

======= [Exercise 4554]

Fermions in magnetic field, quantum phase transition

A d dimensional container ($d = 1, 2, 3$) contains fermions of density n, temperature $T = 0$, mass m and spin $\frac{1}{2}$, having a magnetic moment $\bar{m}$. The container is placed in a magnetic field $H/\bar{m}$ so that the fermion spectra is $\epsilon_{\mathbf{p}} = \frac{\mathbf{p}^2}{2m} \pm H$ where $\mathbf{p}$ is the momentum. (Note that orbital effects are neglected, possible e.g. at d=2 with the field parallel to the layer).

(a) Evaluate the chemical potential $\mu(H)$, for small H: Consider first an expansion to lowest order in H and then evaluate $d\mu/dH$ to note the change at finite H.

(b) Beyond which H_c does the consideration in (a) fail? Find $\mu(H)$ at $H > H_c$ and plot qualitatively $\mu(H)/\mu_0$ as function of H/μ_0 (where $\mu_0 = \mu(H = 0)$) for $d = 1, 2, 3$, indicating the values of $\mu(H)/\mu_0$ at H_c.

(c) Of what order is the phase transition at H_c, at either $d = 1, 2, 3$? Does the phase transition survive at finite T? (no need for finite T calculations – just note analytic properties of thermodynamic functions).

(d) The container above, called A, with $H \neq 0$ is now attached to an identical container B (same fermions at density n, $T = 0$), but with $H = 0$. In which direction will the fermions flow initially? Specify your answer for $d = 1, 2, 3$ at relevant ranges of H.