# Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed
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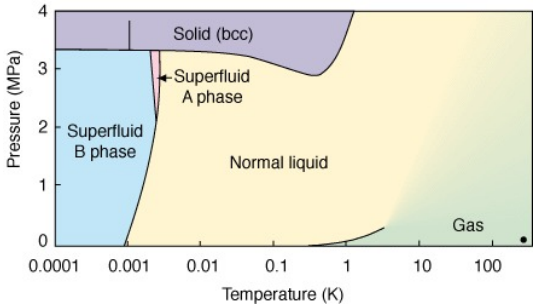
This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

## ═══ [Exercise 4552]

**Liquid He3, cooling by adiabatic pressurization**

The phase-diagram of $^3$He is displayed in the figure below. Assume that the liquid phase can be described as a low temperature ideal Fermi gas with $T_F \approx 5\,^\circ$K, while the solid phase is described as a set of spins. Assume that $T > 0.01\,^\circ$K, such that antiferromagnetic order is diminished and the spins can be treated as non-interacting. In this region the $P(T)$ coexistence solid-liquid curve exhibits a minimum at temperature $T^*$. This implies an anomaly: as the temperature is raised, keeping the pressure constant, the liquid is "frozen" into solid, and only later melts back into a liquid. This is the regime of interest for this question.




In particular we are interested in method of cooling by pressurization. Consider $N$ Helium particles in the the liquid phase with temperature lower than $T^*$. The system is pressurized adiabatically. Once the liquid becomes a solid, the temperature drops down to a lower value.

(a) Express the entropy $S_\ell(T,P)$ in the liquid phase and the entropy $S_s(T)$ in the solid phase. Determine the temperature $T^*$ for which both phases have the same entropy.

(b) Use Clapeyron's relation to deduce the shape of the $P(T)$ coexistence solid-liquid curve near $T^*$. Assume that $P^* \equiv P(T^*)$ is known. Assume that the difference $\Delta v$ of the specific volumes is temperature independent and that the solid is denser.

(c) Of what order is the liquid-solid transition? Evaluate the jump in the specific heat.

(d) Find the trajectory $T(P)$ for an adiabatic process in the liquid phase region, starting at some initial state $(T_0, P_0)$. Assuming that $(T^*, P^*)$ are known, determine the condition that this process leads to cooling via solidification.

(e) If the condition in (d) is satisfied, what will happen to the $T(P)$ trajectory after it hits the $P(T)$ coexistence curve?