

Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed
Department of Physics, Ben-Gurion University, Beer-Sheva 84105, Israel

This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

===== [Exercise 4550]

Equilibrium of Fermions and condensed Bosons $F \rightleftharpoons A+B$

N fermions of type F that have spin $1/2$ are placed in a box of volume V . Each fermion can decay into a boson of type B that have spin 0 , and a fermion of type A that has spin $1/2$. The reaction is $F \rightarrow A + B$, and it has an energy gain ϵ_0 . This means that $A + B$ has a lower binding energy than F . The masses of the particles are m_F, m_A, m_B respectively.

- (1) Assuming ideal gases at temperature T , write the chemical equilibrium condition that determine the densities n_F, n_A, n_B at equilibrium.
- (2) Write the chemical equilibrium condition at $T = 0$. Describe the dependence of the densities on ϵ_0 . Find ϵ_c such that for $\epsilon_0 > \epsilon_c$ the number of F fermions vanishes.
- (3) Assume that the condensation of bosons B occurs at T_c such that $T_c \ll p_F^2 / (2m_A) < \epsilon_0$, where p_F is the Fermi momentum of fermions A . Evaluate T_c and rewrite the condition on T_c in terms of the given parameters.