Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed
Department of Physics, Ben-Gurion University, Beer-Sheva 84105, Israel

This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

======= [Exercise 4546]

Quantum statistics of reactions

(a) The following reaction occurs inside a star

$$\gamma + \gamma \leftrightarrow e^+ + e^-$$

where γ is a photon and $e^\pm$ are the positron and electron, respectively. Assume overall charge neutrality and that the system is in equilibrium at temperature T. Find an expression for the densities of $e^\pm$. (In general $e^\pm$ with mass m are relativistic). Find an explicit result for these densities in the limit $k_B T << mc^2$. (Hint: no conservation law for photons).

(b) Repeat (a) for the reaction

$$\gamma + \gamma \leftrightarrow \pi^+ + \pi^-$$

where $\pi^\pm$ are bosons with mass m_π. Can these bosons become Bose-condensed if the temperature is sufficiently lowered? What are the densities at $T = 0$?

(c) The photons in (b) are now eliminated from the system, but the density of the bosons is maintained. If these bosons are cooled, is there a temperature at which Bose condensation is possible? [Assume that the initial temperature satisfies $k_B T << m_\pi c^2$]. What are the densities at $T = 0$?

(d) Consider initial equal densities $\bar{n}$ for e^+ and for e^-. The $e^\pm$ are allowed to equilibrate with $\pi^\pm$ via the reaction

$$e^+ + e^- \leftrightarrow \pi^+ + \pi^-$$

Write equations that determine the chemical potentials at $T \neq 0$. Consider now $T = 0$ and determine if Bose Einstein condensation is possible and if so what is the condition for that. Consider both $m_e > m_\pi$ and $m_e < m_\pi$ situations.