

Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed
Department of Physics, Ben-Gurion University, Beer-Sheva 84105, Israel

This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

===== [Exercise 4450]

Beta decay $n \rightarrow p + e + \bar{\nu}$

In this problem we regard (anti)neutrino as massless spin 1/2 fermions. If we place neutrons in some confined region, some of them β -decay into protons, electrons and antineutrinos via the reaction $n \leftrightarrow p + e^- + \bar{\nu}$. The masses are m_n, m_p, m_e . The objective is to estimate the final equilibrium density of n of the neutrons, given that that initially density was n_0 .

- (a) Find the equations of states for gas of massless spin 1/2 fermions. Namely, express the density of particles and the density of energy as a function of the chemical potential μ , the temperature T , and the speed of light c . Evaluate the integrals for $T = 0$ and for large T . Define what does it mean large T .
- (b) Write equations for the neutron density n , given that initially there were only neutrons with density n_0 , and the temperature is T . Consider approximation under the following assumptions:
 - (b1) The particles are non-relativistic (except the neutrino)
 - (b2) The particles are hyper relativistic (negligible mass).
 - (b3) The temperature is zero.
 - (b4) The temperature is high (Boltzmann approximation).