Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

====== [Exercise 4213]

Chemical equilibrium for A==A+e

N_0 atoms of type A are placed in an empty box of volume V, such that their initial density is $n_0 = N_0/V$. The ionization energy of the atoms is ε_0. The box is held in temperature T, and eventually a chemical equilibrium $A \rightleftarrows A^+ + e^-$ is reached. The fraction of ionized atoms is $x = N^+/N_0$. The masses of the particles are m_e for the electron, and $m_{A^+} \approx m_A$ for the atoms and the ions.

(1) Define temperature T_0 such that $T \gg T_0$ is a sufficient condition for treating the gas of atoms in the Boltzmann approximation.

(2) Assuming the Boltzmann approximation for both the atoms and the electrons, write an equation for x. Write its *approximate* solution assuming $x \ll 1$. Write the condition for the validity of the latter assumption.

(3) Assuming that $x \ll 1$, write a condition on the density n_0, that above T_0 it was legitimate to treat the electrons in the Boltzmann approximation. Note: the condition is a simple inequality and should be expressed using $(m_e, m_A, \varepsilon_0)$.

Assume that the condition in (3) breaks down. It follows that there is a regimes $T_0 \ll T \ll T_1$ where the atoms can be treated in the Boltzmann approximation, while the electrons can be treated as a low temperature quantum gas.

(4) Write an equation for x assuming that the electrons can be treated approximately as a zero temperature Fermi gas. Exotic functions should not appear. You are not expected to solve this transcendental equation.

(5) What would be the equation for x if the electrons were Bosons instead of Fermions.

Note: Express all the final answers using $(m_e, m_A, n_0, \varepsilon_0, T)$, and *elementary* functions.
Exotic functions should not appear. It is allowed to use the notation $\lambda_e(T) = (2\pi/\mathsf{m}_e T)^{1/2}$.