$\boxed{AS2}$ (HW 2009 ...)

Law of Mass Action

$\nu C \longleftrightarrow A + B$ $\qquad$ $(\underbrace{2HD}_{C} \leftrightarrow \underbrace{H_2}_{A} + \underbrace{D_2}_{B}$ דוטריום$)$

$\nu \mu_C = \mu_A \quad (\mu_A = \mu_B) \rightarrow \frac{\partial}{\partial \mu_C} = \nu \frac{\partial}{\partial \mu_A} \rightarrow N_C = \frac{\partial F}{\partial \mu_C} = \nu \frac{\partial F}{\partial \mu_A} = \nu N_A$

$$\boxed{dN_C = -\nu \, dN_A \, , \quad dN_A = dN_B}$$

$$Z = \frac{f_A^{N_A}}{N_A!} \cdot \frac{f_B^{N_B}}{N_B!} \cdot \frac{f_C^{N_C}}{N_C!}$$

פונק' החלוקה...

$F = -kT \ln Z$

$$dF = 0 \rightarrow d\ln Z = 0 = d\left[N_A(\ln f_A - \ln N_A + 1) + N_B(\ln f_B - \ln N_B + 1) + N_C(\ln f_C - \ln N_C + 1)\right]$$

$$0 = dN_A(\ln f_A - \ln N_A) + dN_B(\ln f_B - \ln N_B) + dN_C(\ln f_C - \ln N_C)$$

$$= dN_A\left[\ln f_A - \ln N_A + \ln f_B - \ln N_B - \nu \ln f_C + \nu \ln N_C\right]$$

$$\rightarrow \nu \ln \frac{f_C}{N_C} = \ln \frac{f_A}{N_A} + \ln \frac{f_B}{N_B}$$

$$\rightarrow \left(\frac{f_C}{N_C}\right)^{\nu} = \frac{f_A}{N_A} \cdot \frac{f_B}{N_B} \rightarrow \left(\frac{f_C}{n_C}\right)^{\nu} = V^{\nu - 2} \frac{f_A f_B}{n_A n_B}$$

$$\rightarrow \boxed{\frac{n_C^{\nu}}{n_A n_B} = V^{2-\nu} \frac{f_C^{\nu}}{f_A f_B}}$$

b) $$E_{n,l} = \frac{p^2}{2m} + \underset{\hbar\omega_0}{\underset{\shortparallel}{h\nu}}\left(n+\tfrac{1}{2}\right) + \frac{l(l+1)h^2}{8\pi^2 I}$$

$$f_x = \frac{1}{\lambda^3} \sum_n e^{-\beta h\nu(n+\frac{1}{2})} \sum_l e^{-\beta\frac{l(l+1)h^2}{8\pi^2 I}} \cdot \underbrace{(2l+1)}_{\text{degeneracy}} \cdot$$

see (*)

$$= \frac{1}{\lambda^3}\, \frac{e^{-\frac{1}{2}\beta h\nu}}{1-e^{-\beta h\nu}} \cdot \frac{8\pi^2 I}{\beta h^2} \cdot \text{const} \cdot \left(\frac{1}{2}\right)$$

if A=B half of the degrees of freedom

$$\sim m_x^{3/2} \cdot \frac{I}{\nu}$$

We mark $\underset{\substack{\text{translational}\\\text{(momentum related)}}}{m_t = m_A+m_B}$ $\quad \underset{\text{reduced}}{m_r = \frac{m_A m_B}{m_A+m_B}}$, And we have

$$I = m_r \cdot r_r^2 \qquad \nu \sim m_r^{-1/2} \Leftrightarrow \left(\omega \sim \sqrt{\tfrac{k}{m_r}}\right) \to \frac{I}{\nu} \sim m_r^{3/2}$$

$$f \sim m_t^{3/2} \cdot m_r^{3/2} = (m_A m_B)^{3/2}$$

$$K = \frac{f_{HD}^2}{f_{H_2} f_{D_2}} = \frac{(m_A m_D)^3}{\underbrace{(m_A^2)^{3/2}\cdot\frac{1}{2}}_{f_{H_2}} \cdot \underbrace{(m_D^2)^{3/2}\cdot\frac{1}{2}}_{f_{D_2}}} = 4$$

See (*)

$$\sum_l (2l+1)\, e^{-\frac{\beta l(l+1)h^2}{8\pi^2 I}} \xrightarrow[\text{(classical limit)}]{} \int_0^\infty d[l(l+1)]\, e^{-\frac{\beta h^2}{8\pi^2 I}\cdot[l(l+1)]} = \frac{8\pi^2 I}{\beta h^2}$$