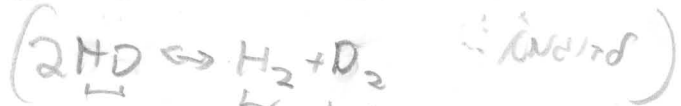


AS2 (2009 4.4)

Law of Mass Action



$V \mu_c = \mu_A \quad (\mu_A = \mu_B) \rightarrow \frac{\partial}{\partial \mu_c} = V \frac{\partial}{\partial \mu_A} \rightarrow N_c = \frac{\partial F}{\partial \mu_c} = V \frac{\partial F}{\partial \mu_A} = V N_A$

$dN_c = -V dN_A, \quad dN_A = dN_B$

... ..

$Z = \frac{\rho_A^{N_A}}{N_A!} \cdot \frac{\rho_B^{N_B}}{N_B!} \cdot \frac{\rho_C^{N_C}}{N_C!}$

$F = -kT \ln Z$

$dF = 0 \rightarrow d \ln Z = 0 = d \left[N_A (\ln \rho_A - \ln N_A + 1) + N_B (\ln \rho_B - \ln N_B + 1) + N_C (\ln \rho_C - \ln N_C + 1) \right]$

$0 = dN_A (\ln \rho_A - \ln N_A) + dN_B (\ln \rho_B - \ln N_B) + dN_C (\ln \rho_C - \ln N_C)$

$= dN_A \left[\ln \rho_A - \ln N_A + \ln \rho_B - \ln N_B - V \ln \rho_C + V \ln N_C \right]$

$\rightarrow V \ln \frac{\rho_C}{N_C} = \ln \frac{\rho_A}{N_A} + \ln \frac{\rho_B}{N_B}$

$\rightarrow \left(\frac{\rho_C}{N_C} \right)^V = \frac{\rho_A}{N_A} \cdot \frac{\rho_B}{N_B} \rightarrow \left(\frac{\rho_C}{N_C} \right)^V = V^{V-2} \frac{\rho_A \rho_B}{N_A N_B}$

$\rightarrow \frac{N_C^V}{N_A N_B} = V^{2-V} \frac{\rho_C^V}{\rho_A \rho_B}$

$$b) E_{n,l} = \frac{p^2}{2m} + \hbar \nu \left(n + \frac{1}{2}\right) + \frac{l(l+1)\hbar^2}{8\pi^2 I}$$

$$\rho_x = \frac{1}{\lambda^3} \sum_n e^{-\beta \hbar \nu \left(n + \frac{1}{2}\right)} \sum_l e^{-\frac{\beta l(l+1)\hbar^2}{8\pi^2 I}} \cdot (2l+1)$$

$\hbar \nu_0$ \downarrow \downarrow
 see (*) degeneracy

$$= \frac{1}{\lambda^3} \frac{e^{-\frac{1}{2}\beta \hbar \nu}}{1 - e^{-\beta \hbar \nu}} \cdot \frac{8\pi^2 I}{\beta \hbar^2} \cdot \text{const} \cdot \left(\frac{1}{2}\right)$$

$$\sim m_x^{3/2} \cdot \frac{I}{\nu}$$

\uparrow if $A=B$
 half of the degrees of freedom

We mark $m_T = m_A + m_B$ translational (momentum related) $m_r = \frac{m_A m_B}{m_A + m_B}$ reduced, and we have

$$I = m_r \cdot r^2 \quad \nu \sim m_r^{-1/2} \left(\omega \sim \sqrt{\frac{k}{m_r}} \right) \rightarrow \frac{I}{\nu} \sim m_r^{3/2}$$

$$\rho \sim m_T^{3/2} \cdot m_r^{3/2} = (m_A m_B)^{3/2}$$

$$K = \frac{\delta H_0^2}{\delta n_i \delta \rho_2} = \frac{(m_A m_D)^3}{\underbrace{(m_A^2)^{3/2} \cdot \frac{1}{2}}_{\delta H_2} \cdot \underbrace{(m_D^2)^{3/2} \cdot \frac{1}{2}}_{\delta \rho_2}} = 4$$

see (*)

$$\sum_l (2l+1) e^{-\frac{\beta l(l+1)\hbar^2}{8\pi^2 I}} \xrightarrow{\text{(classical limit)}} \int_0^\infty d[l(l+1)] e^{-\frac{\beta \hbar^2}{8\pi^2 I} \cdot [l(l+1)]} = \frac{8\pi^2 I}{\beta \hbar^2}$$