Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

═══ [Exercise 4017]

Adsorbsion and fractal dimension

Surfactant Adsorption: A dilute solution of surfactants can be regarded as an ideal three dimensional gas. As surfactant molecules can reduce their energy by contact with air, a fraction of them migrate to the surface where they can be treated as a two dimensional ideal gas. Surfactants are similarly adsorbed by other porous media such as polymers and gels with an affinity for them.

(a) Consider an ideal gas of classical particles of mass m in d dimensions, moving in a uniform potential of strength ϵ_d. Show that the chemical potential at a temperature T and particle density n_d, is given by

$$\mu_d = \epsilon_d + k_B T \ln[n_d \lambda^d] \qquad \text{where} \qquad \lambda = \frac{h}{\sqrt{2\pi m k_B T}}$$

(b) If a surfactant lowers its energy by ϵ_0 in moving from the solution to the surface, calculate the concentration of coating surfactants as a function of the solution concentration n (at $d = 3$).

(c) Gels are formed by cross-linking linear polymers. It has been suggested that the porous gel should be regarded as fractal, and the surfactants adsorbed on its surface treated as a gas in d_f dimensional space, with a non-integer d_f . Can this assertion be tested by comparing the relative adsorption of surfactants to a gel, and to the individual polymers (assuming it is one dimensional) before cross-linking, as a function of temperature?