4016

$$H = \frac{p_\theta^2}{2I} + \frac{p_\beta^2}{2I\sin^2\theta} - \mathcal{E} a\cos\theta \qquad I = m a^2$$

$$Z_1 = \frac{I}{2\pi\beta}\iint d\Omega\, e^{\beta\mathcal{E}a\cos\theta}$$ See Ex 2173

For perp. Field $\quad Z_1^{\perp} = \frac{I}{\beta}\,\frac{e^{\beta\mathcal{E}a}-1}{\beta\mathcal{E}a}$

For parallel $\quad Z_1^{\parallel} = \frac{I}{\beta}\,\sinh c(\beta\mathcal{E}a)$

$$\mathcal{Z}(\beta,\mu,\mathcal{E}) = \left[1 + z_1(\beta,\mathcal{E})\, e^{\beta\mu}\right]^M$$

$$N = \frac{1}{\beta}\frac{\partial \ln \mathcal{Z}}{\partial \mu} = \frac{M}{1 + (1/z_1)\, e^{-\beta\mu}}$$

$$D = \frac{1}{\beta}\frac{\partial \ln \mathcal{Z}}{\partial \mathcal{E}} = N \cdot \frac{1}{\beta}\frac{\partial \ln z_1}{\partial \mathcal{E}}$$

$$N = \frac{M}{1 + (ma^2\tau)^{-1}\, e^{-\mu/\tau}} \qquad (\mathcal{E} \to 0)$$

$$D(\mathcal{E}) \approx N \cdot \begin{cases} \frac{1}{2}a \\ \frac{1}{3}\frac{a^2}{\tau}\mathcal{E} \end{cases}$$

$$z_1^{\perp} \approx 1 + \tfrac{1}{2}(\beta\mathcal{E}a)$$

$$z_1^{\parallel} \approx 1 + \tfrac{1}{6}(\beta\mathcal{E}a)^2$$