

A43 Grand Canonically

$$Z_{\text{pole}} = \int_0^{\pi} d\theta \sin\theta \cdot e^{\beta E d \cos\theta} = \frac{e^{\beta E d} - 1}{\beta E d} = Z(\beta E d)$$

$$Z_1 = 1 + e^{\beta \mu} Z_{\text{dipole}} = 1 + e^{\beta \mu} \frac{e^{\beta E d} - 1}{\beta E d} = 1 + e^{\beta \mu} Z(\beta E d)$$

$$Z_M = Z_1^M$$

הסתברות ממוצעת - $\langle \cos\theta \rangle$ קוונטום

$$N = -\frac{\partial \ln Z_M}{\partial \mu} = kT \frac{\partial \ln Z_M}{\partial \mu} = kT M \cdot \frac{\partial \ln Z_1}{\partial \mu}$$

$$\frac{N}{M} = kT \cdot \frac{\beta \cdot e^{\beta \mu} \frac{e^{\beta E d} - 1}{\beta E d}}{1 + e^{\beta \mu} \frac{e^{\beta E d} - 1}{\beta E d}} = \frac{1}{1 + e^{-\beta \mu} \frac{1}{Z(\beta E d)}} = \frac{N}{M} \xrightarrow{E \rightarrow 0} \frac{1}{1 + e^{-\beta \mu}}$$

Fermi-Dirac

$\lim_{E \rightarrow 0} Z(\beta E d) = 1$ הסתברות ממוצעת

b

הסתברות ממוצעת

$$\langle d \cos\theta \rangle = d \langle \cos\theta \rangle = \frac{d}{Z_1} \int_0^{\pi} d\theta \sin\theta \cdot \cos\theta \cdot e^{\beta E d \cos\theta} = kT \frac{\partial \ln Z_1}{\partial E} = -\frac{\partial \ln Z_1}{\partial E}$$

$$= kT \frac{e^{\beta \mu}}{1 + e^{\beta \mu} Z(\beta E d)} \cdot \frac{\partial}{\partial E} \left(\frac{e^{\beta E d} - 1}{\beta E d} \right)$$

$$= kT \frac{e^{\beta \mu}}{1 + e^{\beta \mu} \frac{e^{\beta E d} - 1}{\beta E d}} \cdot \left(\frac{-e^{\beta E d} + 1 + e^{\beta E d} E \beta d}{\beta d E^2} \right)$$

$$\langle d \cos\theta \rangle \Big|_{E \rightarrow 0} = kT \cdot \frac{e^{\beta \mu}}{1 + e^{\beta \mu}} \cdot (\beta d) \Big|_{E \rightarrow 0}$$

$$\langle d \cos\theta \rangle_M \Big|_{E \rightarrow 0} = d M \frac{1}{1 + e^{-\beta \mu}} = d \langle N \rangle \Big|_{E \rightarrow 0}$$

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