Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

═══ [Exercise 4016]

Polar adsorption of particles to a surface

Consider an M site system in an equilibrium with gas of particles that have mass m. The chemical potential of the gas is μ and its temperature is T. A particle can bind to a site. Each site can absorb at most one atom. The binding energy is ε, and the length of the bonds is a. In such state it behaves as a rotor that has moment of inertia $I = ma^2$, and a dipole moment qa. The polarization can be in any direction away from the surface (2π staradians).

Tip: The kinetic part in a rotor Hamiltonian is

$$\frac{1}{2I}\left[p_\theta^2 + \frac{p_\varphi^2}{\sin^2(\theta)}\right]$$

(1) Calculate the partition function $Z_\perp(\beta, f)$ for an occupied site, assuming electric field f perpendicular to the surface.

(2) Calculate the partition function $Z_\parallel(\beta, f)$ for an occupied site, assuming electric field f parallel to the surface.

(3) Express the M site grand partition function $\mathcal{Z}(\beta, \mu, f)$ in terms of Z. Additionally, write an explicit expression for zero field.

(4) Express the average number N of adsorbed particles in terms of Z. Additionally, write an explicit expression for zero field.

(5) Find a leading order expression for the average polarization D/N for weak perpendicular f.

(6) Find a leading order expression for the average polarization D/N for weak parallel f.

(*) Tip: one can use a shortcut in the calculation of Z, bypassing the integration over the momentum variables.