Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

═══ [Exercise 4015]

Adsorbtion of polar molecules to a surface

A large number n of identical mass m atoms are bounded within a surface that has M adsorbtion centers. Each adsorbtion center can connect one atom, such that a polar molecule AB is created. The dipole moment of each molecule is d, and it can be oriented either vertically (1 possible orientation) or horizontally (4 possible orientations). The binding energy is ϵ_0. Additionally a vertical electric field $\mathcal{E}$ is applied. The interaction energy between the field and the dipole is $-\vec{\mathcal{E}}\cdot\vec{d}$. The polarization of the system is defined via the expression for the work, $dW = -Dd\mathcal{E}$.

(1) Find the canonical partition function $Z_n(\beta)$ of the system.

(2) Derive an expression for the chemical potential $\mu(T;n)$.

(3) Given μ, deduce what is the coverage $\langle n\rangle$.

(4) Re-derive the expression for $\langle n\rangle$ using the grand canonical partition function $\mathcal{Z}(\beta,\mu)$.

(5) Calculate the polarization $D(\mathcal{E})$ of the system.

Remarks: In items (1-2) it is assumed the the system is closed with a given number n of adsorbed atoms. Hence it is treated within the framework of the canonical ensemble. In items (3-4) the system is in equilibrium with a gas of atoms: the chemical potential μ is given, and the average $\langle n\rangle$ should be calculated using the grand-canonical formalism. In item (5) it is requested to verify that the same result is obtained in the canonical and in the grand-canonical treatments.