$\boxed{B12}$ (2009 G:3)

(spin)

$$Z(h) = \frac{1}{N(h)!}\left(\frac{2V}{\lambda^3} e^{-\beta m g h}\right)^{N(h)}$$

a)

$$F(h) = kT N(h)\left[\ln \frac{N(h)\lambda^3}{2} - 1\right] + mgh\, N(h)$$

$$\mu(h) = \frac{\partial F(h)}{\partial N(h)} = kT \ln \frac{n(h)\lambda^3}{2} + mgh$$

$$\mu(h) = \mu(h=0) \rightarrow n(h) = n(0) e^{-\beta m g h}$$

b) $$n = \frac{2}{V}\sum_p \frac{1}{e^{\beta\left(\frac{p^2}{2m} + mgh - \mu\right)} + 1}$$

at $T=0$, $\mu = \varepsilon_F$ and is h independent

at $h=0$: $\varepsilon_F = \frac{\hbar^2}{2m}\left[3\pi^2 n(0)\right]^{2/3}$

at $h>0$: $\varepsilon_F = \frac{p_F^2}{2m} + mgh$ (we "shift" the chem. pot.)

$$\frac{p_F^2(h)}{2m} = \frac{\hbar^2}{2m}\left(3\pi^2 n(h)\right)^{2/3}$$

$$\varepsilon_F - mgh = \frac{p_F^2(h)}{2m}$$

$$\left(3\pi^2 n(0)\right)^{2/3} - \frac{2m^2 g h}{\hbar^2} = \left(3\pi^2 n(h)\right)^{2/3}$$

$n(h) = \dots$

c) Non degenerate: $$kT \approx \frac{p_F^2(h_c)}{2m} = \frac{1}{2m}\left(3\pi^2 n(h_c)\right)^{2/3}$$

$$\approx \frac{1}{2m}\left[3\pi^2 n(0)\right]^{2/3} - mgh_c$$

$$= \varepsilon_F - mgh_c$$

but since $\varepsilon_F \gg kT$ $\quad \varepsilon_F \approx mgh_c \rightarrow \boxed{h_c \approx \frac{\varepsilon_F}{mg}}$

from (a) ($h \gg h_c$) $\quad \mu = kT\ln\left(n(h)\lambda^3\right) + mgh$

at $h=0$ $\quad \mu = \varepsilon_F = \frac{1}{2m}\left[3\pi^2 n(0)\right]^{3/2}$ $\Big\}\rightarrow n(h) = \frac{1}{\lambda^3} e^{-\beta m g h + \beta\frac{\left(3\pi^2 n(0)\right)^{2/3}}{2m}}$