

Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed
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This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horowitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

===== [Exercise 3711]

Fermions in gravitation field of a star

Consider a neutron star as non-relativistic gas of non-interacting neutrons of mass m in a spherical symmetric equilibrium configuration. The neutrons are held together by a gravitational potential $-GMm/r$ of a heavy object of mass M and radius r_0 at the center of the star. G is the gravity constant and r is the distance from the center.

- (a) Give an expression for $n(r)$ at $T > 0$ using Li functions.
- (b) Consider the neutrons as fermions at $T = 0$ and find $n(r)$, for a given $n(r_0)$.
- (c) Calculate it explicitly in the Boltzmann approximation.
- (d) Repeat items (b) and (c) for a general potential $-A/r^\alpha$.
- (e) For $T = 0$, what is the upper bound on $n(r_0)$ and on the total number N of neutrons if the chemical potential is increased towards zero. Distinguish $\alpha > 2$ from $\alpha < 2$.