Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed
Department of Physics, Ben-Gurion University, Beer-Sheva 84105, Israel

This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

======= [Exercise 3710]

Fermions in gravitation field of a star

Consider an artificial model of neutron star where the gas of N neutrons is held together by a gravitational potential $U = -GMm/r$ generated by the solid core M of the star, where G is the gravitation constant, and m is mass of a neutron. The core has a radius $r \sim 0$.

(a) Assume the neutron gas as Fermi gas. Find the density $n(r)$ for $T = 0$, and determine the Fermi energy $\epsilon_f = -W(N)$.

(b) What is the radius R that is occupied by neutrons?

(c) Find the flux of particles that escape the gravitational field assuming a low temperature T. Use the analogy to thermionic emission.

(d) Write a differential equation for the number $N(t)$ of remaining particles, assuming that the temperature is not changing.

(e) The calculation of the flux in item (c) involves a Boltzmann approximation. Write the condition on T for the validity of this assumption. Additionally, point out what is the systematic error that follows from the constant T assumption.

Note: In item (a) your answer depends on a numerical constant C that you have to define in terms of an elementary definite integral.